Deconstructing Data Science

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Info 290 Lecture 8: Naive Bayes

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elements of probability in many of these methods



Random variable

• A variable that can take values within a fixed set (discrete) or within some range (continuous).

$X \in \{1, 2, 3, 4, 5, 6\}$

 $X \in \{the, a, dog, cat, runs, to, store\}$

$$P(X = x)$$

Probability that the random variable X takes the value x (e.g., 1)

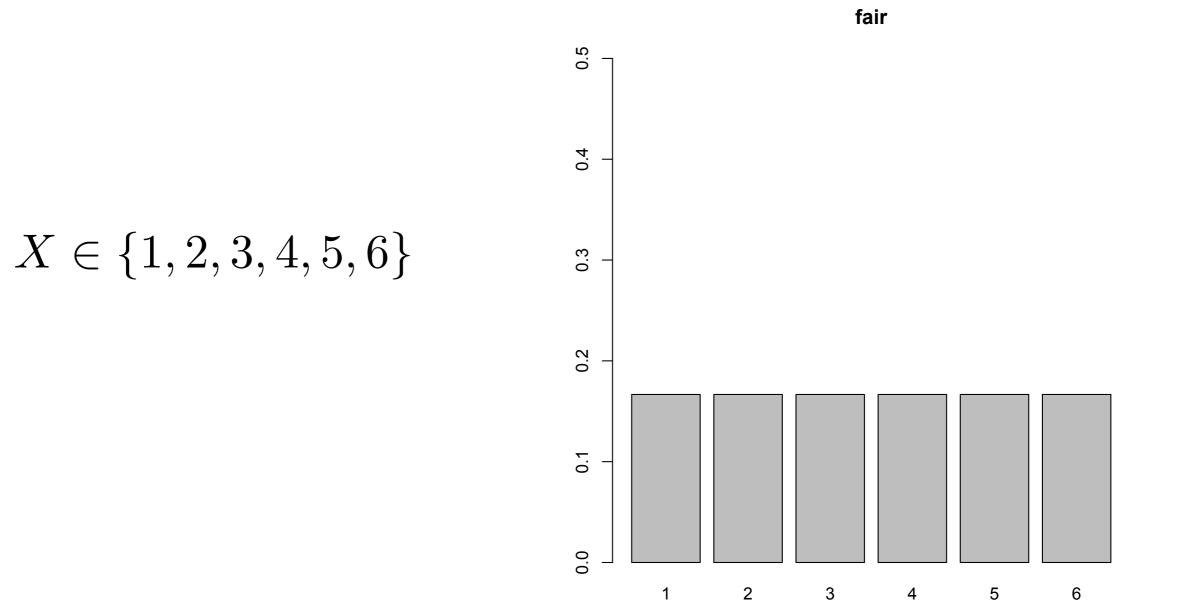
$$X \in \{1, 2, 3, 4, 5, 6\}$$

Two conditions:

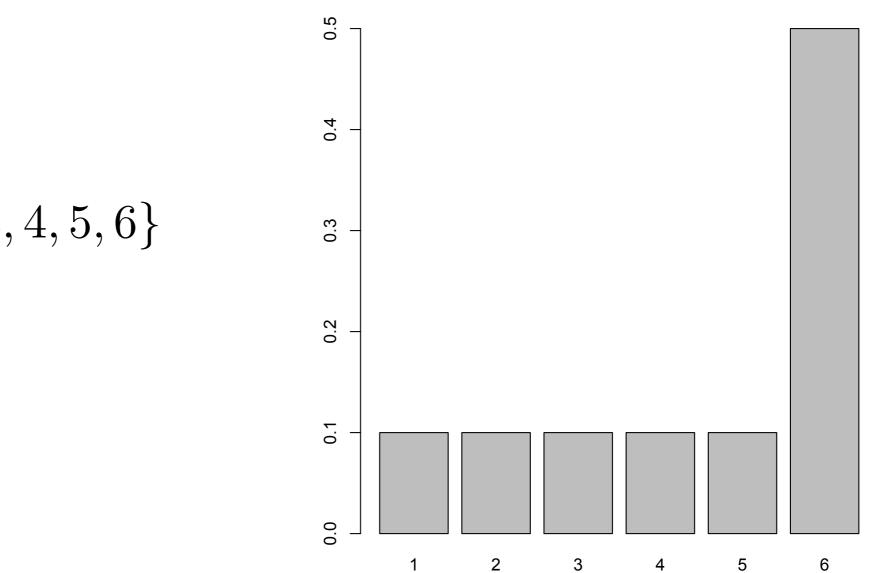
- 1. Between 0 and 1:
- 2. Sum of all probabilities = 1

 $0 \le P(X = x) \le 1$ $\sum_{x} P(X = x) = 1$

Fair dice



Weighted dice



not fair

$X \in \{1, 2, 3, 4, 5, 6\}$

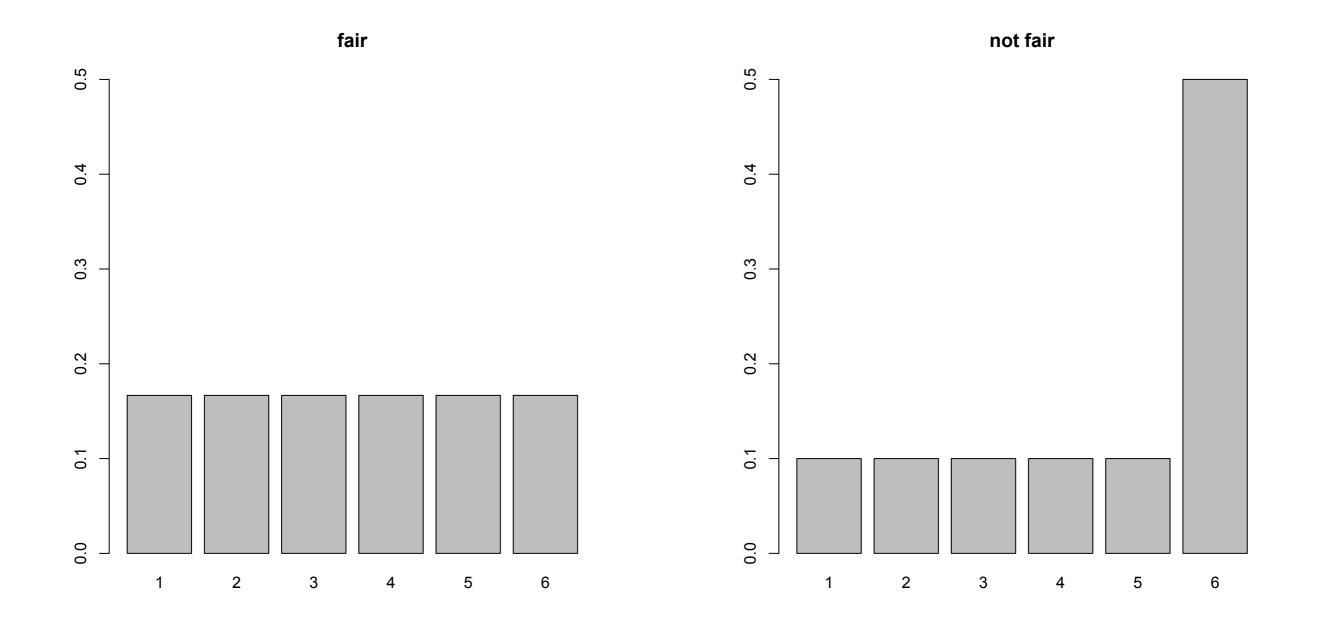
Inference

 $X \in \{1, 2, 3, 4, 5, 6\}$

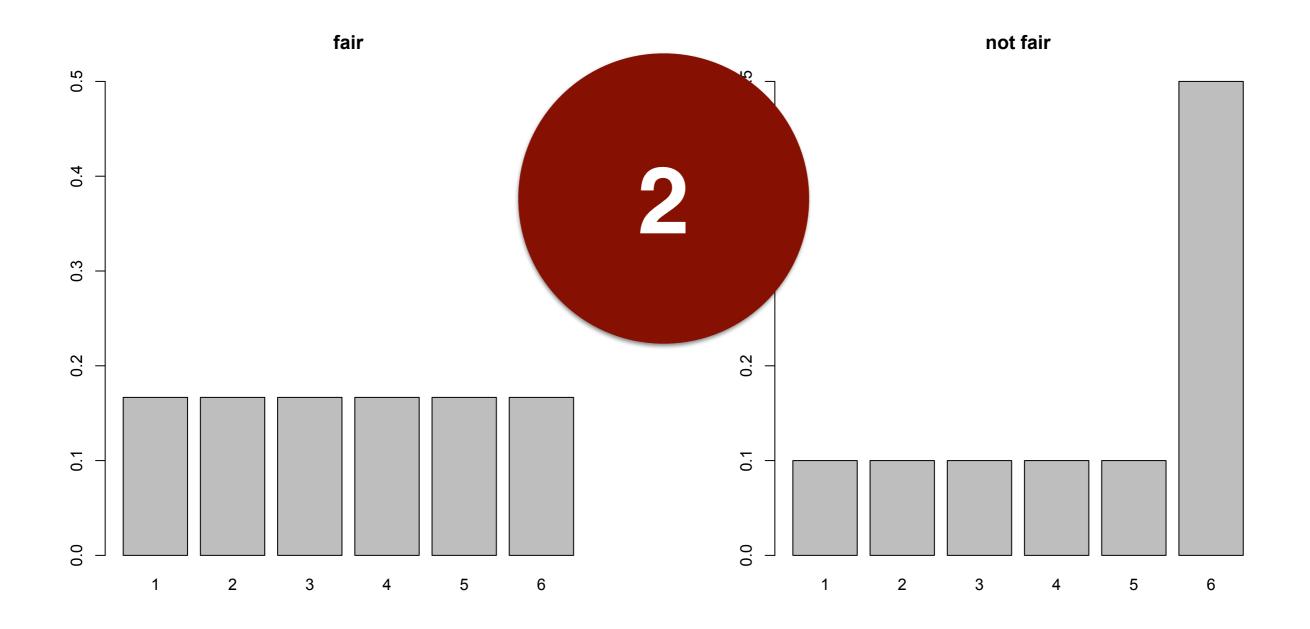
We want to *infer* the probability distribution that generated the data we see.



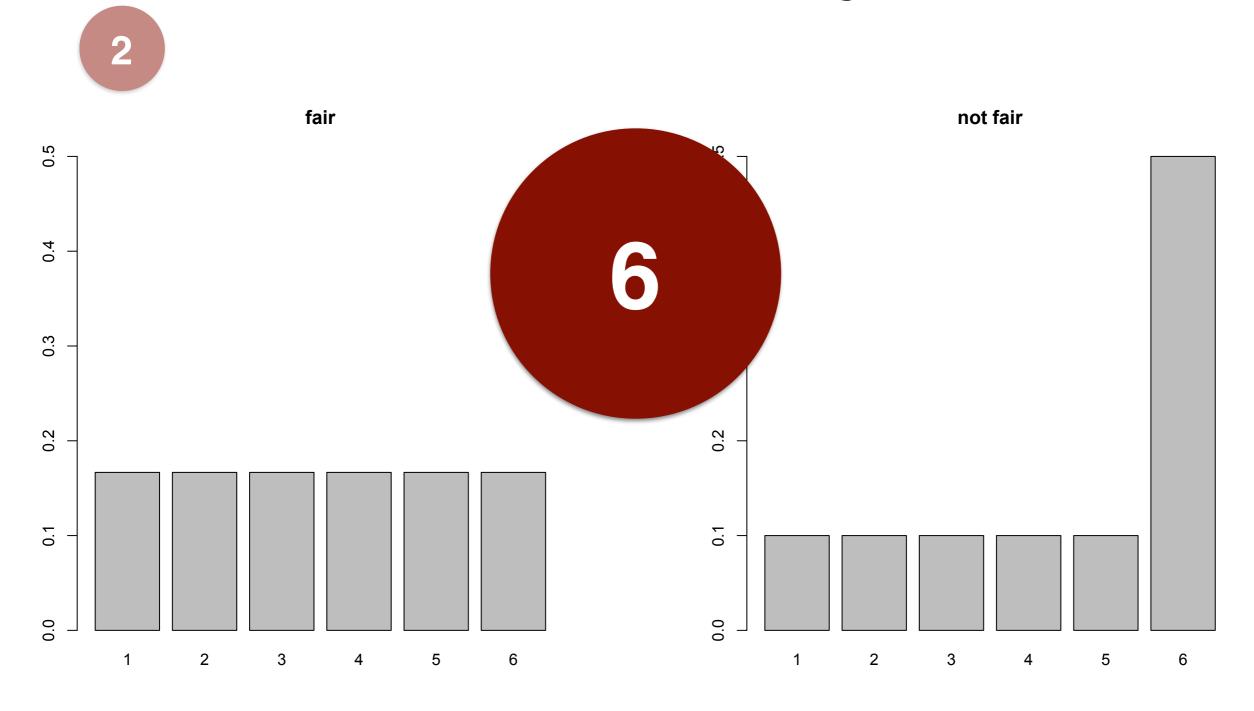
Probability



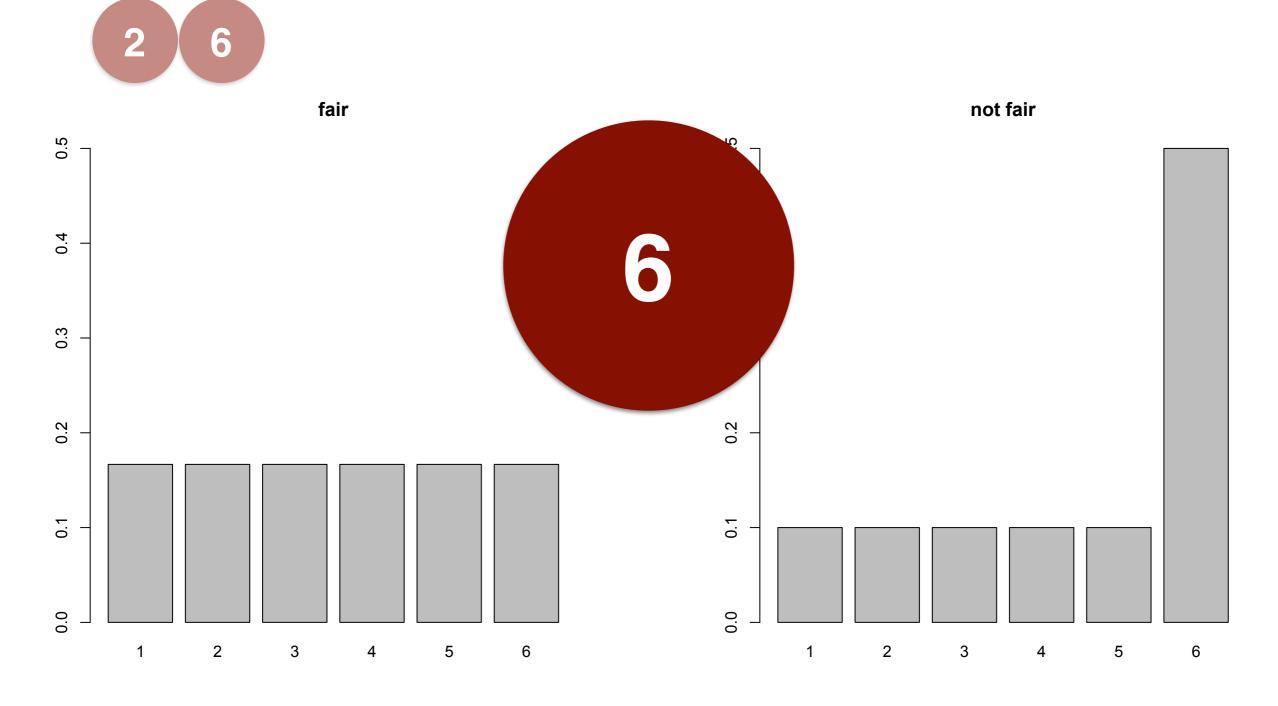
Probability

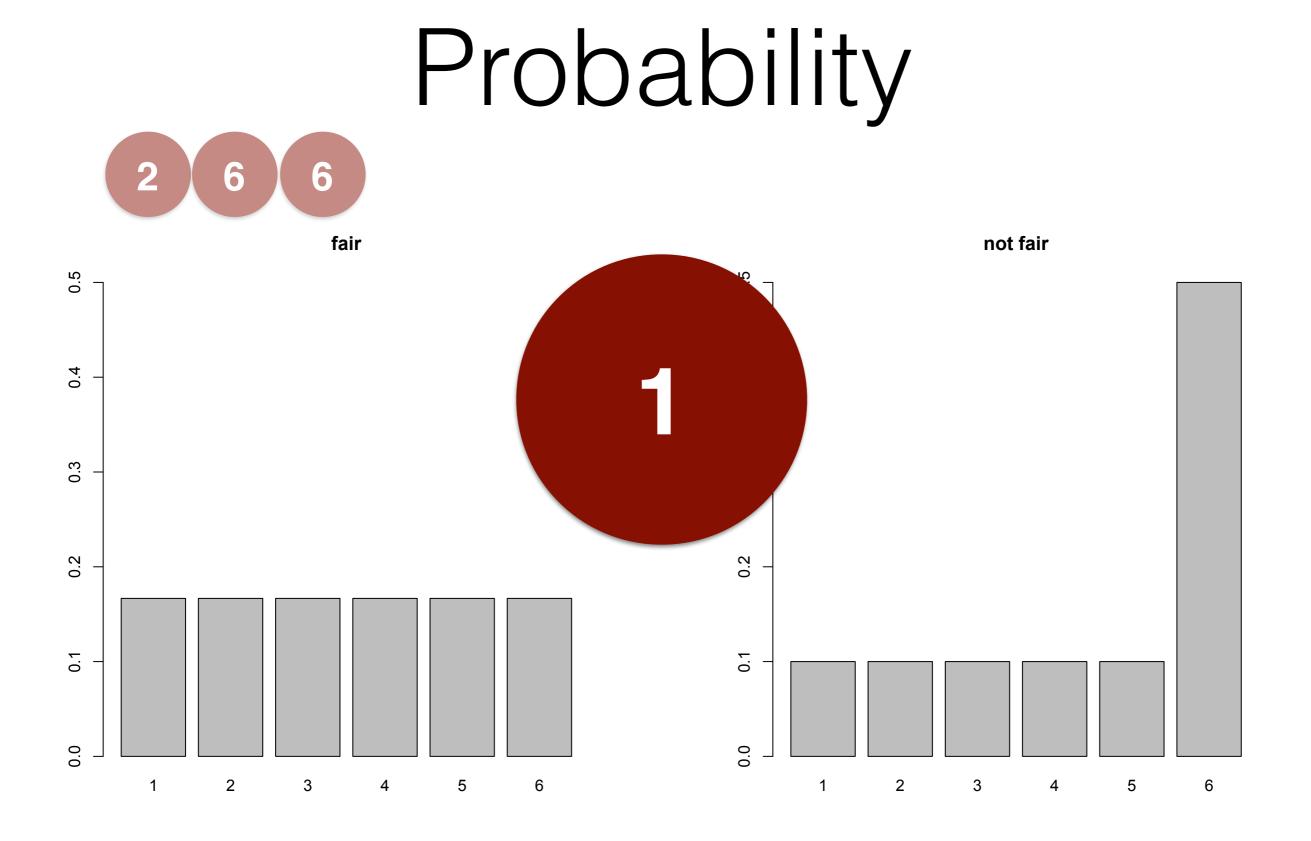


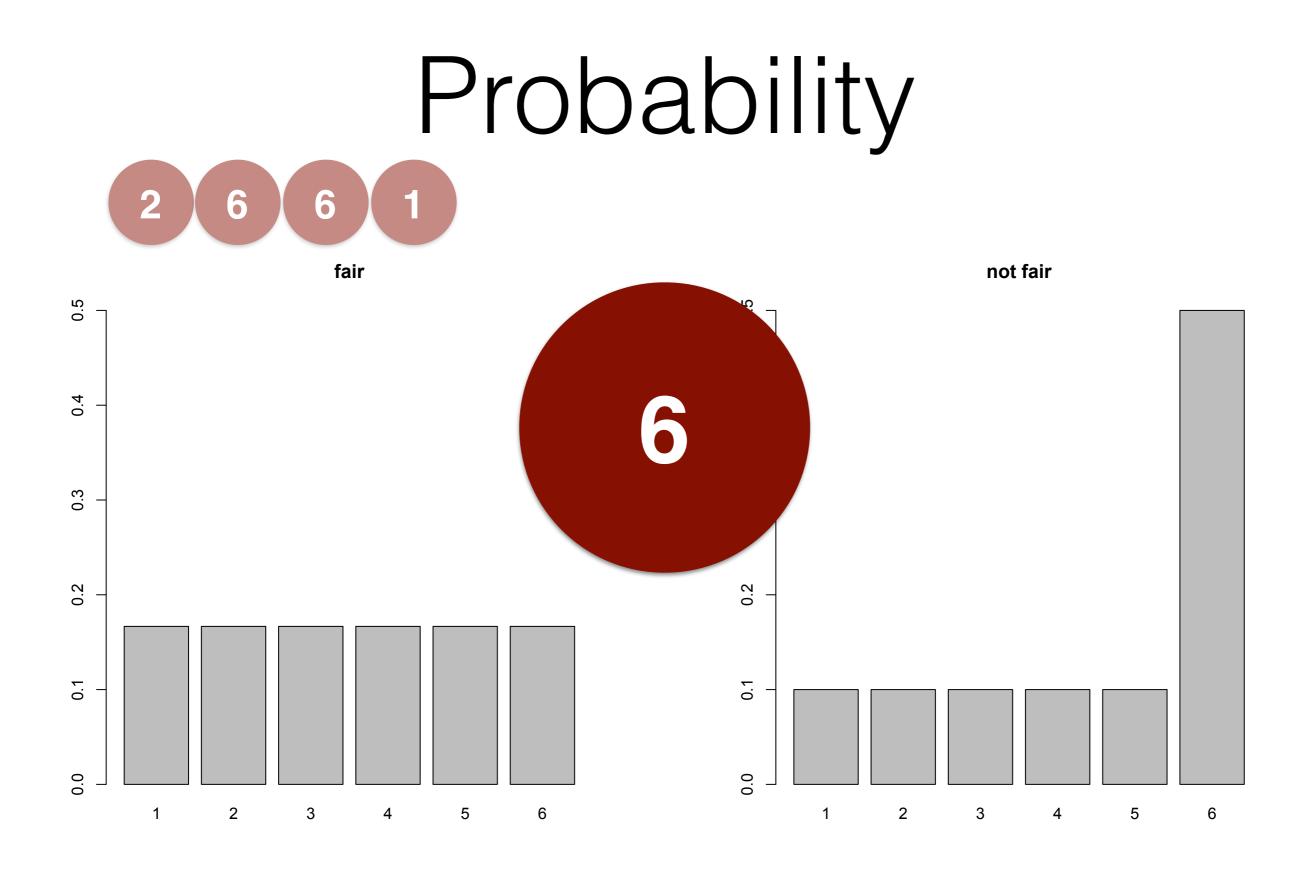
Probability

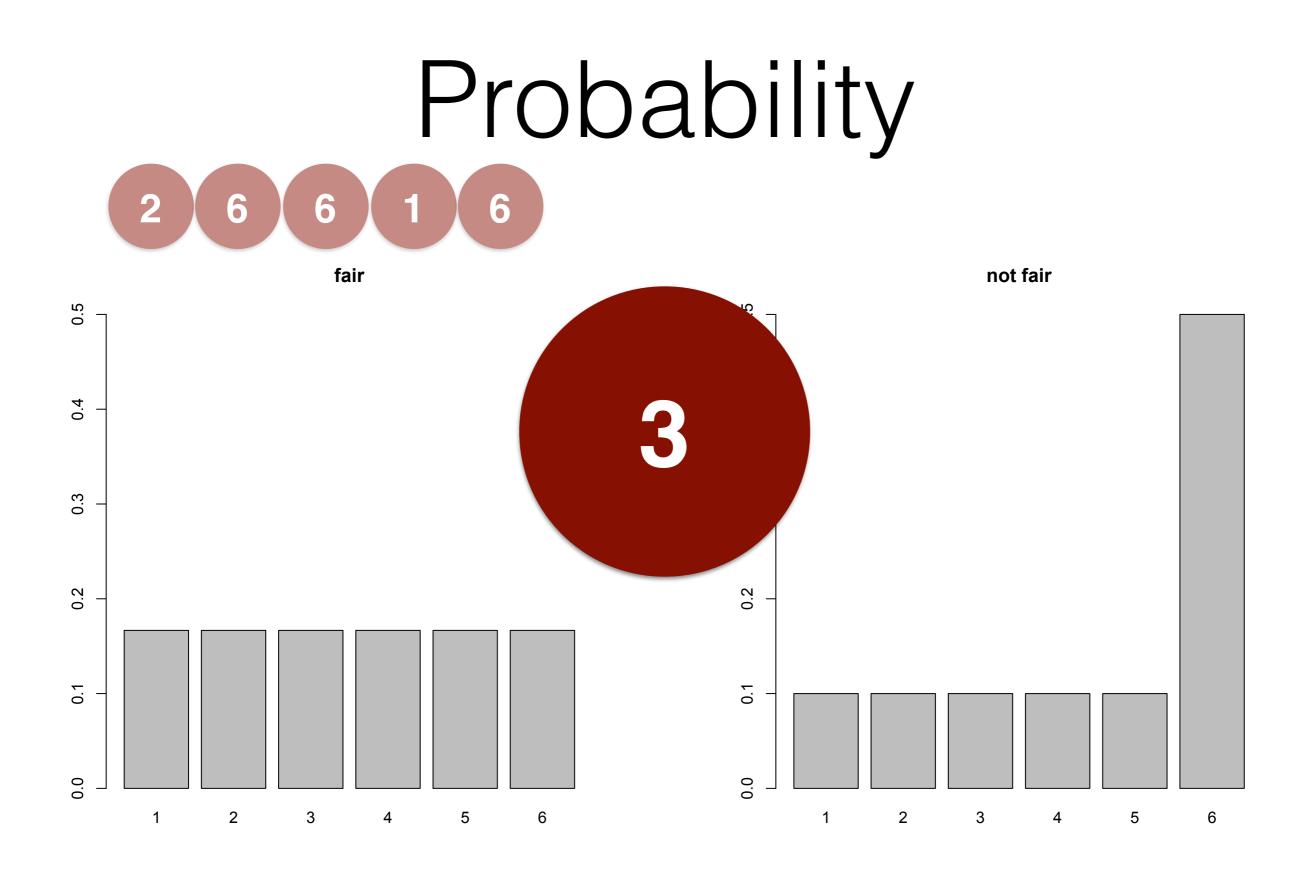


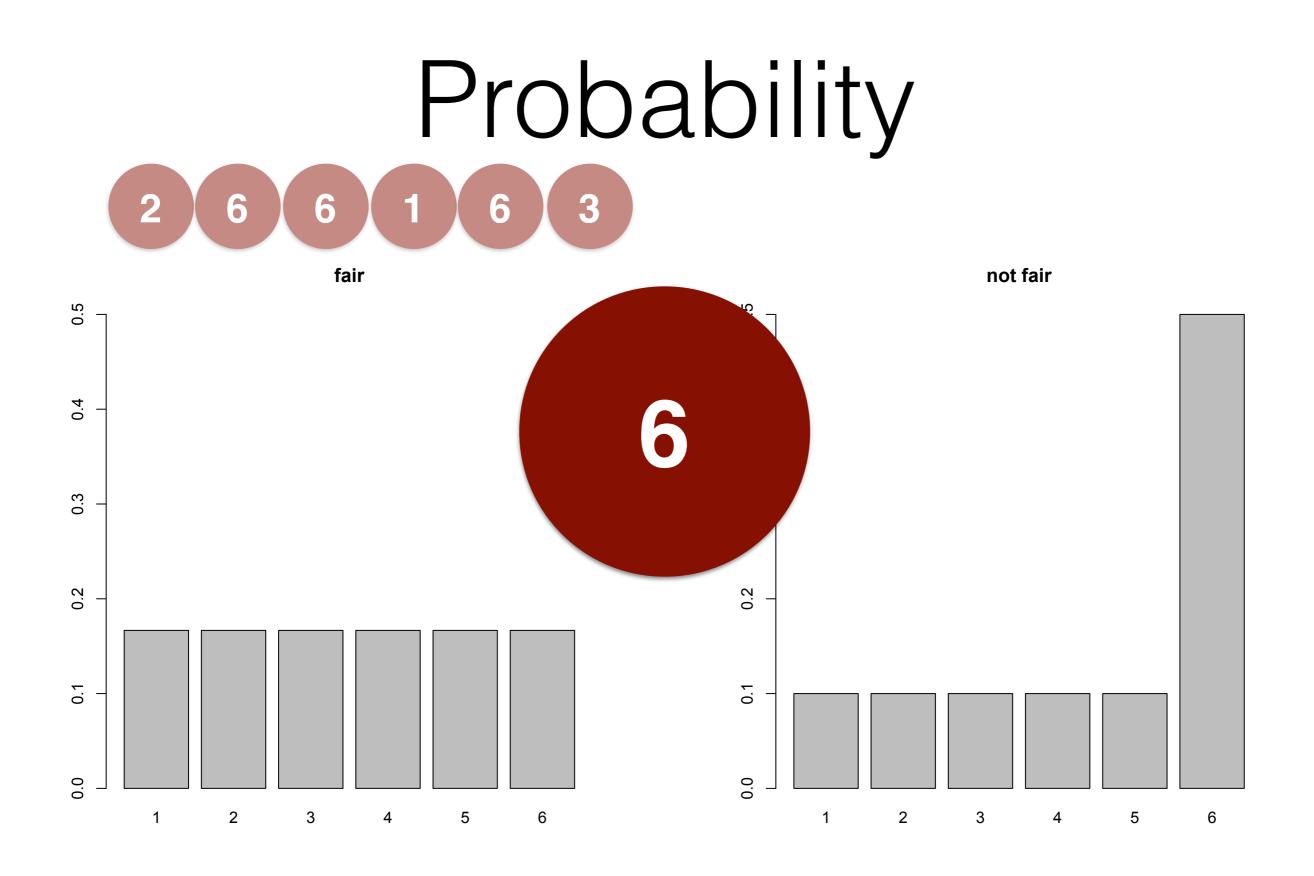
Probability

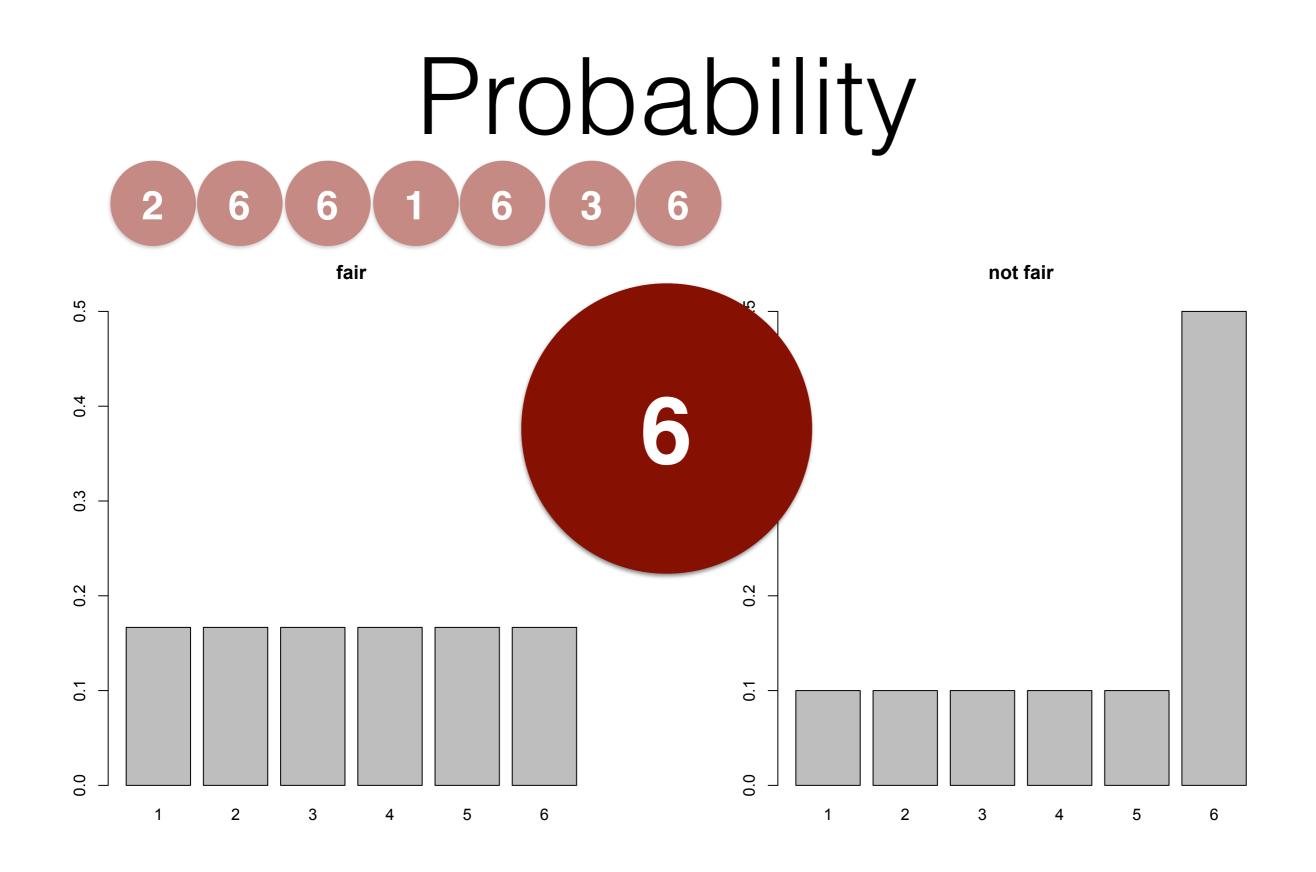


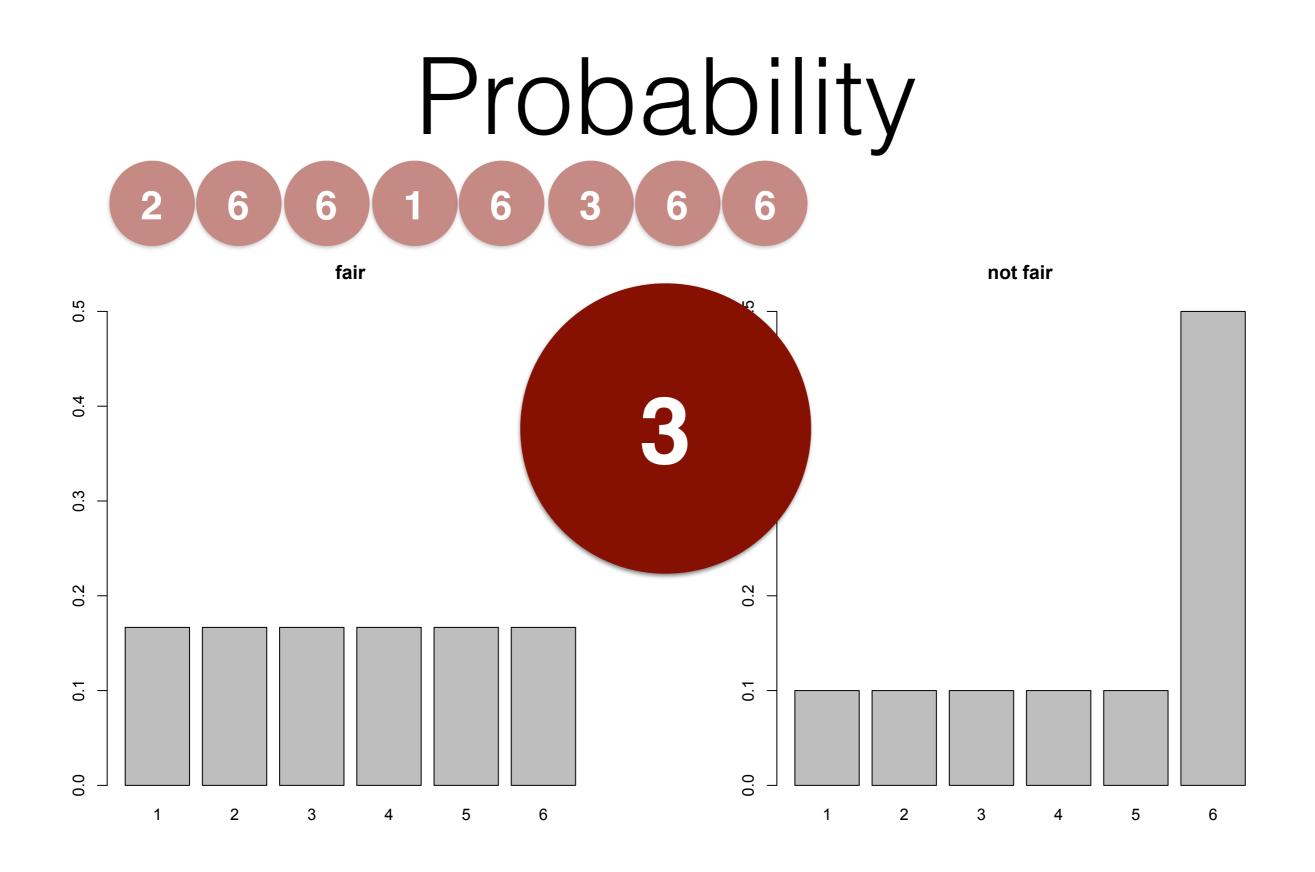


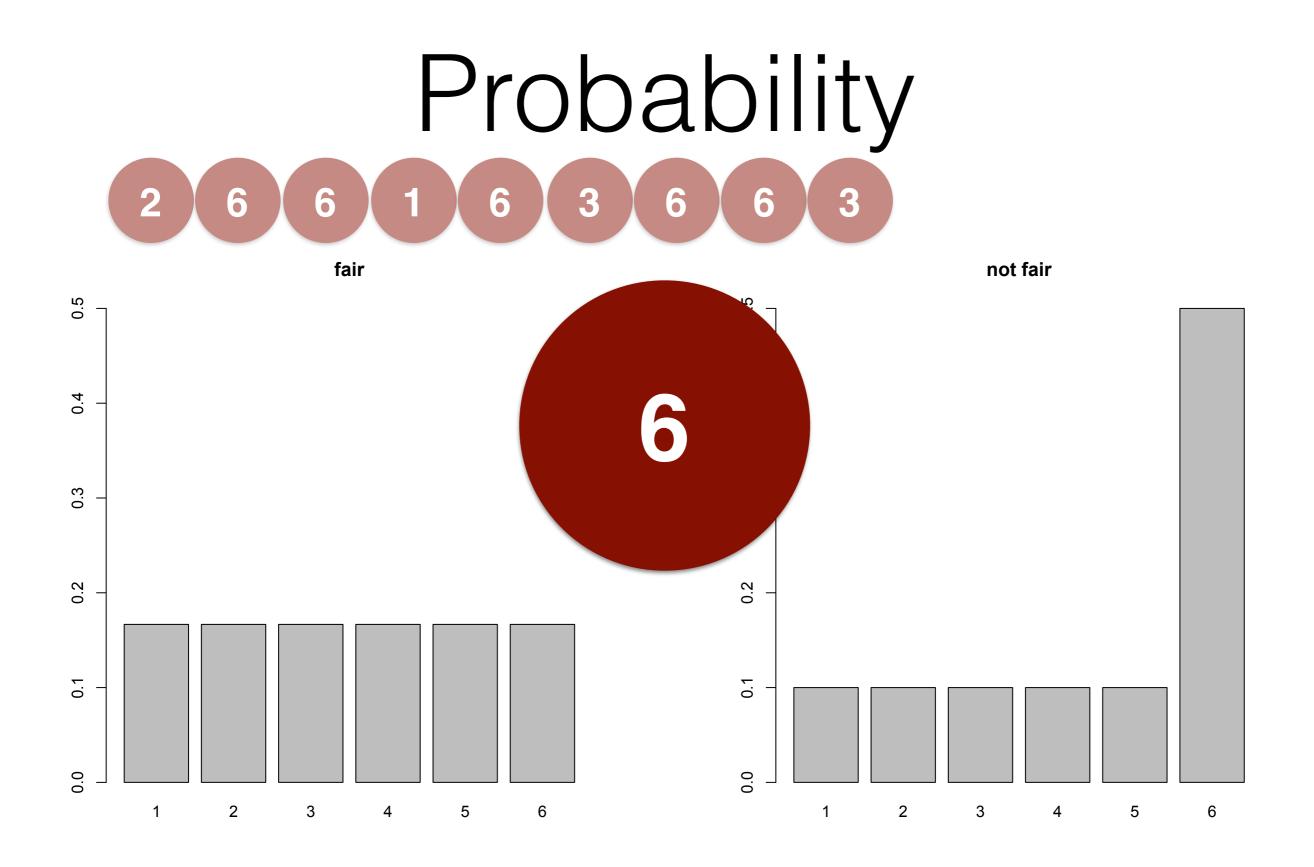


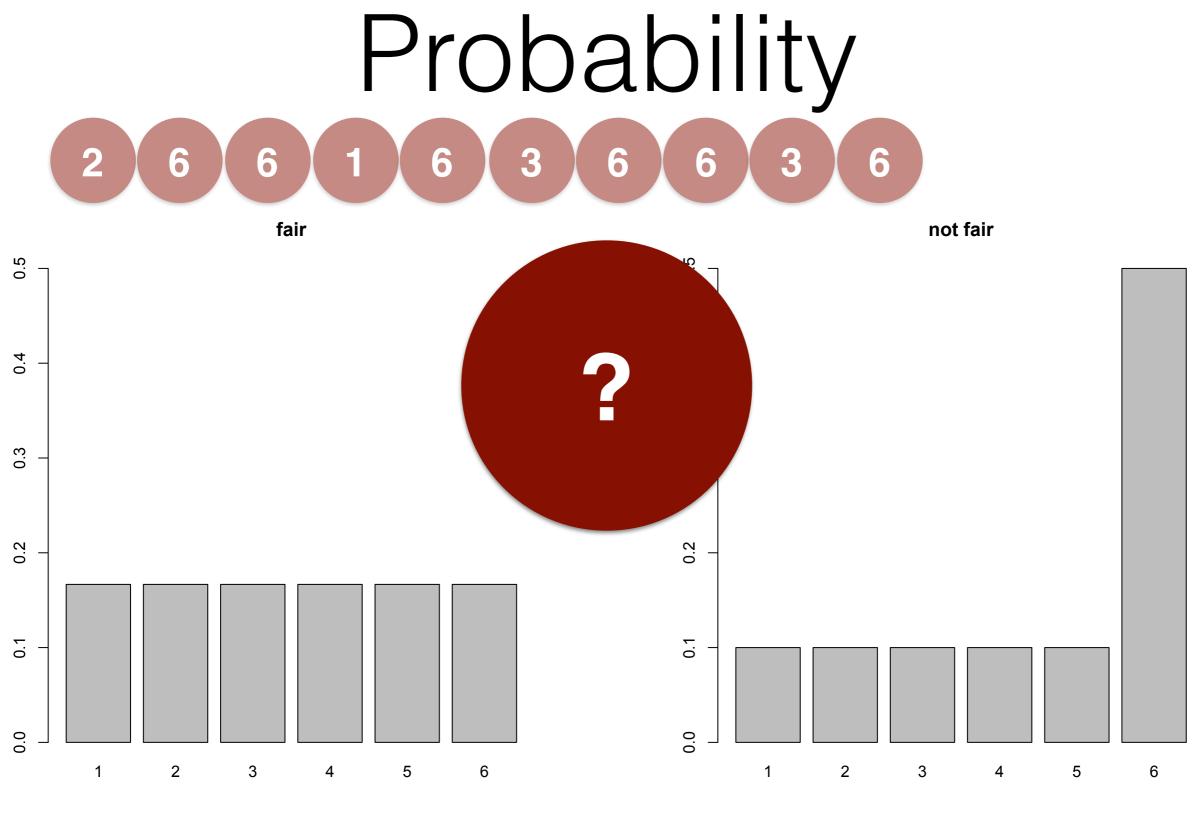












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Independence

• Two random variables are independent if:

$$P(A,B) = P(A) \times P(B)$$

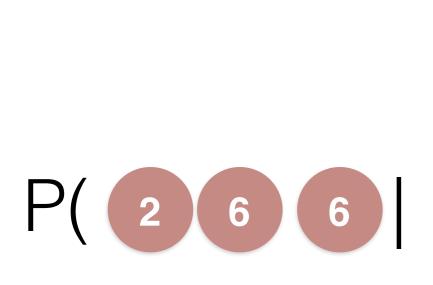
• In general:

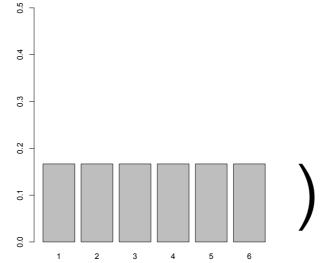
$$P(x_1,\ldots,x_n)=\prod_{i=1}^N P(x_i)$$

 Information about one random variable (B) gives no information about the value of another (A)

$$P(A) = P(A \mid B) \qquad P(B) = P(B \mid A)$$

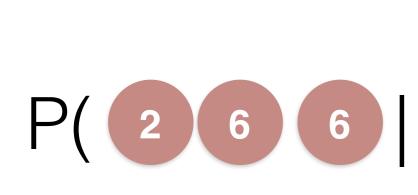
Data Likelihood

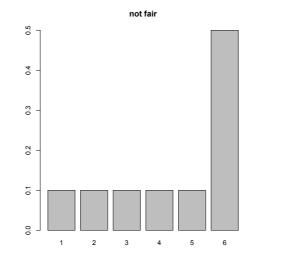




fair

=.17 x .17 x .17 = 0.004913





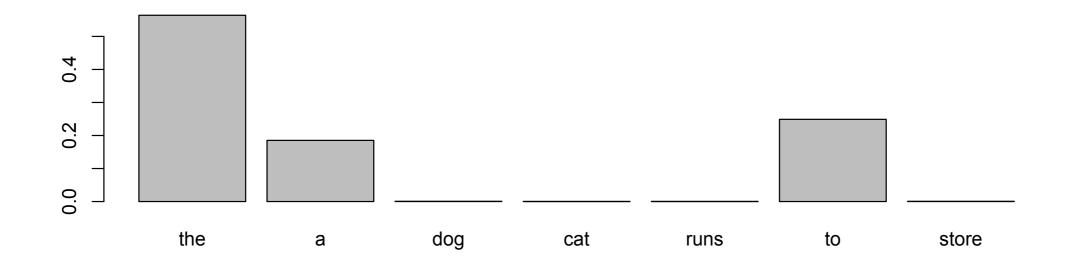
= .1 x .5 x .5 = 0.025

Data Likelihood

 The likelihood gives us a way of discriminating between possible alternative parameters, but also a strategy for picking a single best* parameter among all possibilities

Unigram probability

 $X \in \{the, a, dog, cat, runs, to, store\}$



How do we calculate this?

ined hopes of being admitted to a sight of the young ladies, of whose beauty he had heard much; but ly the father. The ladies were somewhat more fortunate, for they had the advantage of ascertaining frow window that he wore a blue coat, and rode a black horse. An invitation to dinner was soon afterwards where a later of the ladies were somewhat more fortunate, for they had the advantage of ascertaining frow were arrived which deferred it all. Mr. Bingley was obliged to be in town the following day, and, conseq to accept the honour of their invitation, etc. Mrs. Bennet was quite disconcerted. She could not imagination accept the honour of their invitation, etc. Mrs. Bennet was quite disconcerted. She could not imagination accept the honour of their invitation, etc. Mrs. Bennet was quite disconcerted. She could not imagination accept the honour of their invitation, etc. Mrs. Bennet was quite disconcerted. She could not imagination accept the honour of their invitation, etc. Mrs. Bennet was quite disconcerted. She could not imagination accept the honour of their invitation, etc. Mrs. Bennet was quite disconcerted. She could not imagination accept the honour of their invitation, etc. Mrs. Bennet was quite disconcerted. She could not imagination accept the honour of their invitation, etc. Mrs. Bennet was quite disconcerted. She could not imagination accept the honour of their invitation, etc. Mrs. Bennet was quite disconcerted. She could not imagination accept the honour of their invitation, etc. Mrs. Bennet was quite disconcerted. She could not imaginate accept the honour of their invitation, etc. Mrs. Bennet was quite disconcerted. She could not imaginate accept the honour of their invitation, etc. Mrs. Bennet at the following the accept the honour of their invitation, etc. Mrs. Bennet at the following the accept the honour of the accept the interval accept the honour of the accept the honour of the accept the interval accept the honour of the accept the interval accept the honour of the accept the honour of the a

by room it consisted of only five r young man. Mr. Bingley was g ted manners. His sisters were f looked the gentleman; but his f ome features, noble mien, and t

P(X="the") = 28/536 = .052

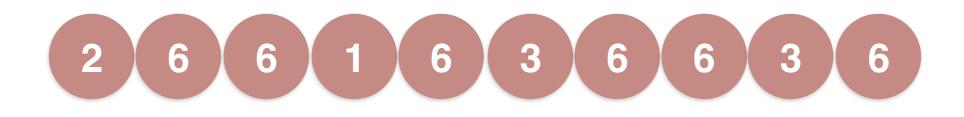
, the husband of the eldest, and d a pleasant countenance, and ion. His brother-in-law, Mr. Hurs of the room by his fine, tall per on within five minutes after his

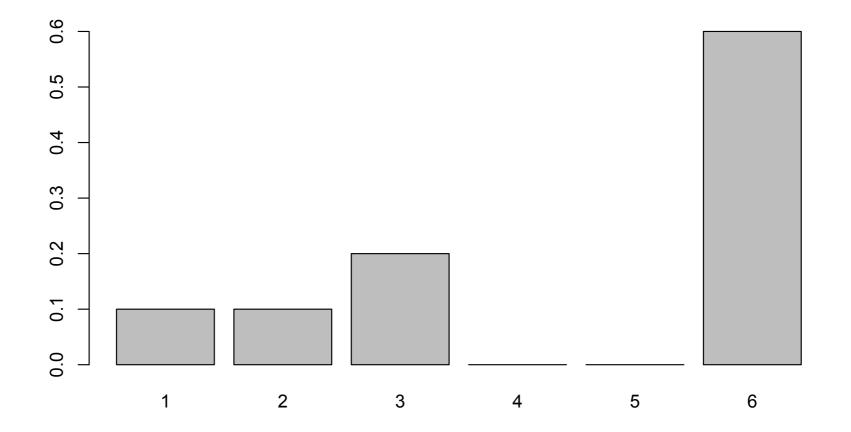
be, of his having ten thousand a year. The gentemen pronounced num to be a fine figure of a man, the ed he was much handsomer than Mr. Bingley, and he was looked at with great admiration for about ha g, till his manners gave a disgust which turned the tide of his popularity; for he was discovered to be p bove his company, and above being pleased; and not all his large estate in Derbyshire could then sa aving a most forbidding, disagreeable countenance, and being unworthy to be compared with his frie gley had soon made himself acquainted with all the principal people in the room; he was lively and rved, danced every dance, was angry that the ball closed so early, and talked of giving one himself a field. Such amiable qualities must speak for themselves. What a contrast between him and his friend! danced only once with Mrs. Hurst and once with Miss Bingley, declined being introduced to any other ent the rest of the evening in walking about the room, speaking occasionally to one of his own party. H ter was decided. He was the proudest, most disagreeable man in the world, and everybody hoped th never come there again. Amongst the most violent against him was Mrs. Bennet, whose dislike of his

Maximum Likelihood Estimate

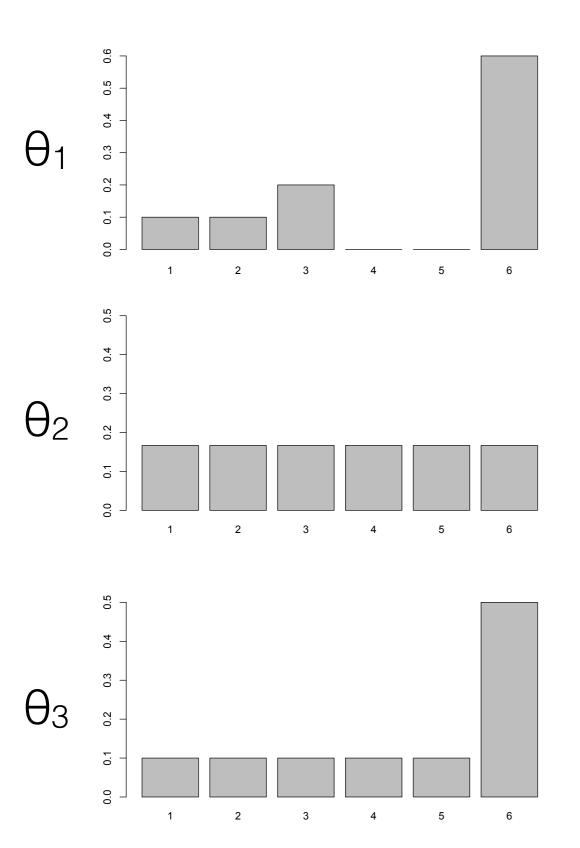
 This is a maximum likelihood estimate for P(X); the parameter values for which the data we observe (X) is most likely.

Maximum Likelihood Estimate









$P(X | \theta_1) = 0.0000311040$

$P(X | \theta_2) = 0.000000992$ (313x less likely)

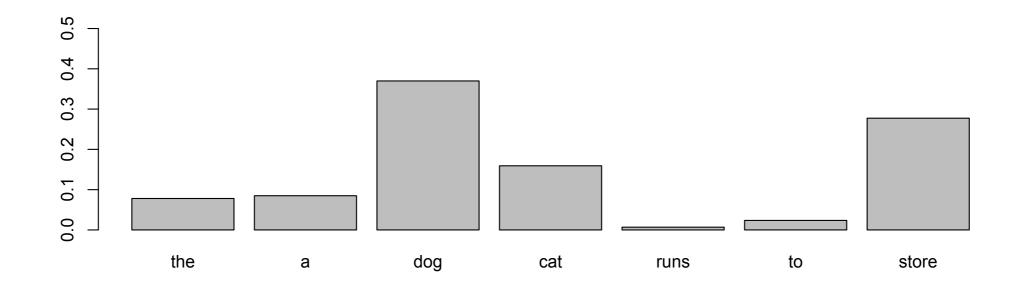
 $P(X | \theta_3) = 0.0000031250$ (10x less likely)

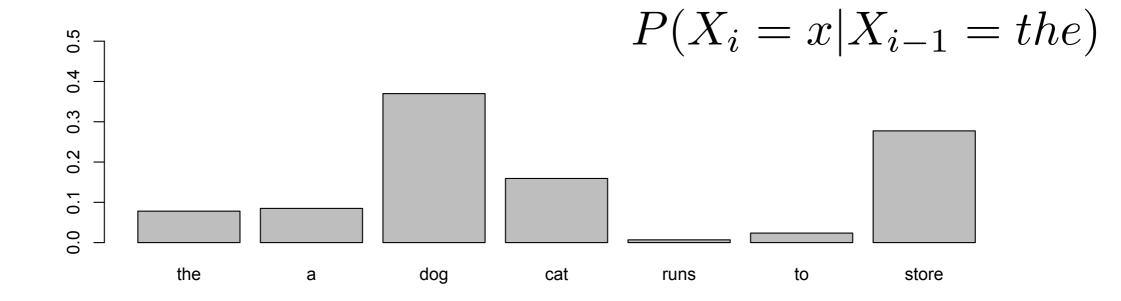
$$P(X = x | Y = y)$$

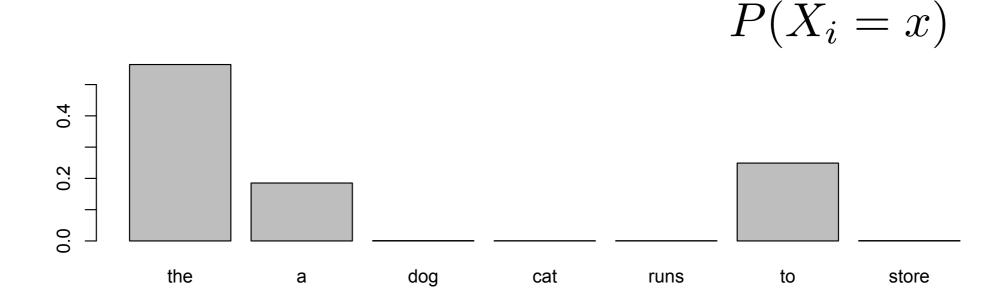
 Probability that one random variable takes a particular value *given* the fact that a different variable takes another

$$P(X_i = dog|X_{i-1} = the)$$

 $P(X_i = dog|X_{i-1} = the)$







ined hopes of being admitted to a sight of the young ladies, of whose beauty he had heard much; buly the father. The ladies were somewhat more fortunate, for they had the advantage of ascertaining f window that he wore a blue coat, and rode a black horse. An invitation to dinner was soon afterwards where a laterady had Mrs. Bennet planned the courses that were to do credit to her housekeeping, were arrived which deferred it all. Mr. Bingley was obliged to be in town the following day, and, quently, unable to accept the honour of their invitation, etc. Mrs. Bennet was quite disconcerted. She agine what business he could have in town so soon after his arrival in Hertfordshire; and she began to might be always flying about from one place to another, and never settled at Netherfield as he ought ucas quieted her fears a little by starting the idea of his being gone to London only to get a large part is and a report soon followed that Mr. Bingley was to bring twelve ladies and seven gentlemen with hir were by. The girls grieved over such a number of ladies, but were comforted the day before the ball by the time of ladies, but were comforted the day before the ball by the time of ladies, but were comforted the day before the ball by the more of ladies, but were comforted the day before the ball by the time of ladies, but were comforted the day before the ball by the time of ladies, but were comforted the day before the ball by the time of ladies, but were comforted the day before the ball by the time of ladies, but were comforted the day before the ball by the time of ladies and seven gentlemen with hir time ball by the time of ladies, but were comforted the day before the ball by the time of ladies.

ty entered the **assembly** ro eldest, and another young m nance, and easy, unaffected -in-law, Mr. Hurst, merely loo y his fine, tall person, hands ive minutes after his entranc

pley, his two sisters, the huse manlike; he had a pleasant in air of decided fashion. His soon drew the **attention** of the ch was in general circulation the pronounced him to be

ure of a man, the **ladies** declared he was much handsomer than Mr. Bingley, and he was looked at wi dmiration for about half the evening, till his manners gave a disgust which turned the **tide** of his popu was discovered to be proud; to be above his company, and above being pleased; and not all his large in Derbyshire could then save him from having a most forbidding, disagreeable countenance, and be hy to be compared with his friend. Mr. Bingley had soon made himself acquainted with all the **princip** in the **room**; he was lively and unreserved, danced every dance, was angry that the **ball** closed so e ked of giving one himself at Netherfield. Such amiable qualities must speak for themselves. What a co en him and his friend! Mr. Darcy danced only once with Mrs. Hurst and once with Miss Bingley, decline introduced to any other lady, and spent the **rest** of the **evening** in walking about the **room**, speaking onally to one of his own party. His character was decided. He was the **proudest**, most disagreeable n rld, and everybody hoped that he would never come there again. Amongst the **most** violent against h ennet, whose dislike of his general behaviour was sharpened into particular resentment by his having

P(X = vampire) vs. P(X = vampire|Y = horror)

P(X = manners | Y = austen) vs. P(X = whale | Y = austen)0.00036 0

P(X = manners | Y = austen) vs. P(X = manners | Y = dickens)0.00036 = 6.7x times more than 0.000053

Authorship Attribution

"Mr. Collins was not a sensible man"





Independence Assumption

"Mr. Collins was not a sensible man" x₁ x₂ x₃ x₄ x₅ x₆ x₇

$P(x_i = Mr., x_2 = Collins) = P(x_i = Mr.) \times P(x_2 = Collins)$

This is certainly untrue in this case, because the presence of Mr. makes Collins more likely (they are dependent)

Independence Assumption

"Mr. Collins was not a sensible man" x₁ x₂ x₃ x₄ x₅ x₆ x₇

We will assume the features are independent:

$$P(x_1, x_2, x_3, x_4, x_6, x_7 \mid c) = P(x_1 \mid c)P(x_2 \mid c) \dots P(x_7 \mid c)$$
$$P(x_i \dots x_n \mid c) = \prod_{i=1}^{N} P(x_i \mid c)$$

A simple classifier

"Mr. Collins was not a sensible man"

Austen		Dickens	
P(X=Mr. Y=Austen)	0.0084	P(X=Mr. Y=Dickens)	0.00421
P(X=Collins Y=Austen)	0.00036	P(X=Collins Y=Dickens)	0.000016
P(X=was Y=Austen)	0.01475	P(X=was Y=Dickens)	0.015043
P(X=not Y=Austen)	0.01145	P(X=not Y=Dickens)	0.00547
P(X=a Y=Austen)	0.01591	P(X=a Y=Dickens)	0.02156
P(X=sensible Y=Austen)	0.00025	P(X=sensible Y=Dickens)	0.00005
P(X=man Y=Austen)	0.00121	P(X=man Y=Dickens)	0.001707

A simple classifier

"Mr. Collins was not a sensible man"

P(X = "Mr. Collins was not a sensible man" | Y = Austen)

= P("Mr" | Austen) × P("Collins" | Austen) × P("was" | Austen) × P("not" | Austen) ... = 0.00000022507322 ($\approx 2.3 \times 10^{-8}$)

P(X = "Mr. Collins was not a sensible man" | Y = Dickens)

P("Mr" | Dickens) × P("Collins" | Dickens) × P("was" | Dickens) × P("not" | Dickens) ... = 0.00000002078906 (≈ 2.1 × 10⁻⁹)

A simple classifier

 The classifier we just specified is a maximum likelihood classifier, where compare the likelihood of the data under each class and choose the class with the highest likelihood

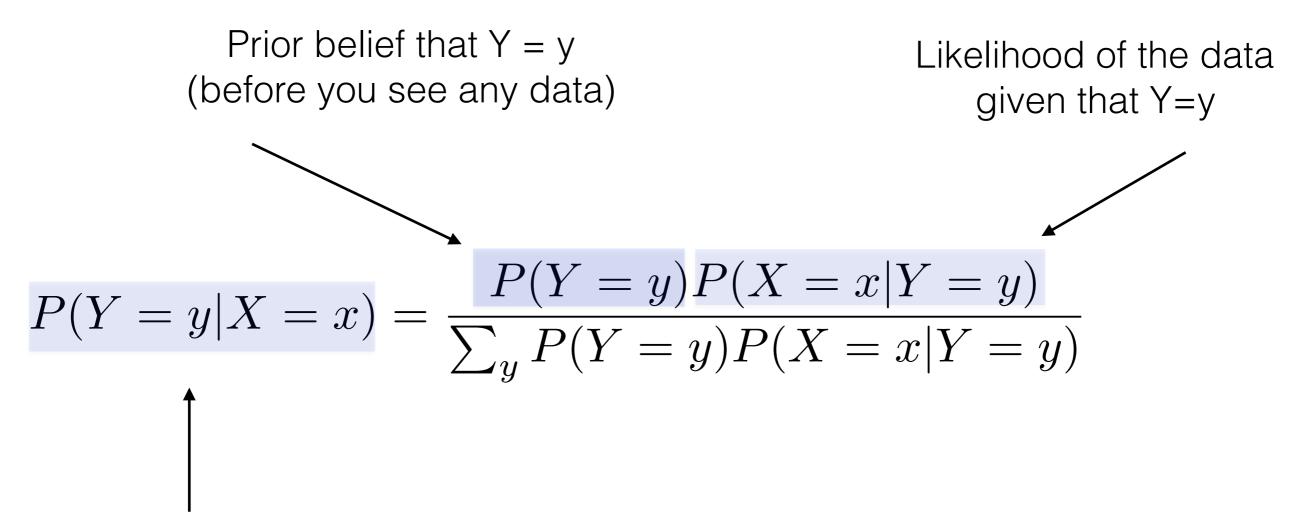
Likelihood: probability of data (here, under class y)

 $P(X = x_i \dots x_n \mid Y = y)$

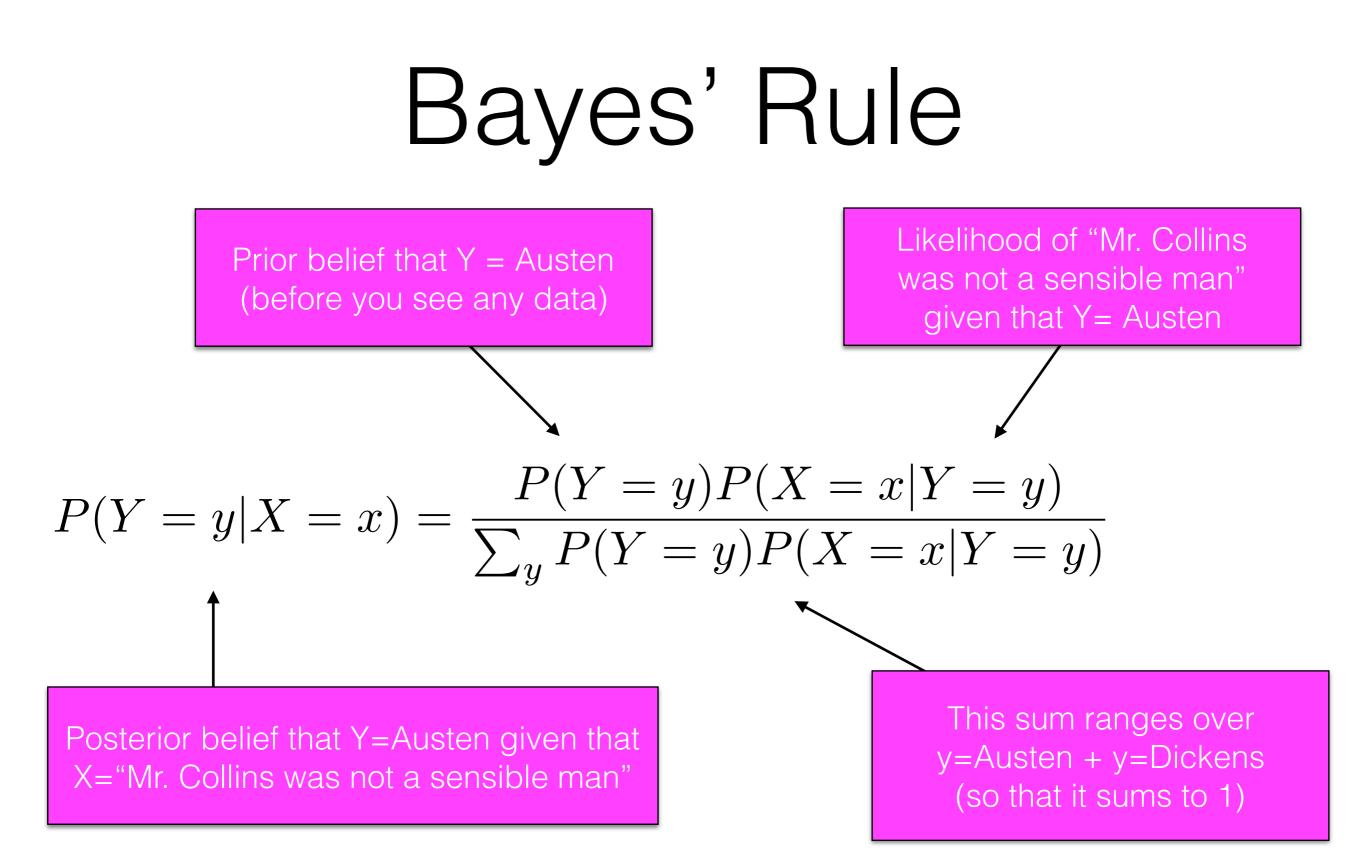
Prior probability of class y

$$P(Y = y)$$





Posterior belief that Y=y given that X=x



Likelihood: probability of data (here, under class y)

Prior probability of class y

Posterior belief in the probability of class y after seeing data

$$P(X = x_i \dots x_n \mid Y = y)$$

P(Y = y)

$$P(Y = y \mid X = x_i \dots x_n)$$

Naive Bayes Classifier

P(Y = Austen)P(X = "Mr..."|Y = Austen)

P(Y = Austen)P(X = "Mr..."|Y = Austen) + P(Y = Dickens)P(X = "Mr..."|Y = Dickens)P(X = "Mr..."

Let's say P(Y=Austen) = P(Y=Dickens) = 0.5 (i.e., both are equally likely a priori)

$$= \frac{0.5 \times (2.3 \times 10^{-8})}{0.5 \times (2.3 \times 10^{-8}) + 0.5 \times (2.1 \times 10^{-9})}$$

$$P(Y = Austen | X = "Mr...") = 91.5\%$$

 $P(Y = Dickens | X = "Mr...") = 8.5\%$

Taxicab Problem

"A cab was involved in a hit and run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following data:

• 85% of the ca		Blue.
 A witness ide the witness u the accident 	Lion't lanoro prior intermation	d the reliability of ed on the night of ly identified each
one of the two	o colors 80% of the time and failed 20%	6 of the time.

What is the probability that the cab involved in the accident was Blue rather than Green knowing that this witness identified it as Blue?"

(Tversky & Kahneman 1981)

Prior Belief

- Now let's assume that Dickens published 1000 times more books than Austen.
 - P(Y= Austen) = 0.000999
 - P(Y = Dickens) = 0.999001

 $\frac{0.000999 \times (2.3 \times 10^{-8})}{0.000999 \times (2.3 \times 10^{-8}) + 0.999001 \times (2.1 \times 10^{-9})}$

P(Y = Austen | X) = 0.011P(Y = Dickens | X) = 0.989

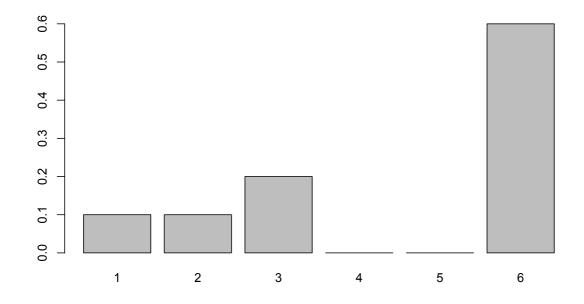
Priors

 Priors can be informed (reflecting expert knowledge) but in practice, but priors in Naive Bayes are often simply estimated from training data

$$P(Y = \text{Austen}) = \frac{\# \text{ of Austen texts}}{\# \text{ of total texts}}$$

Smoothing

 Maximum likelihood estimates can fail miserably when features are never observed with a particular class.



What's the probability of:



Smoothing

 One solution: add a little probability mass to every element.

maximum likelihood estimate

smoothed estimates

$$P(x_i \mid y) = \frac{n_{i,y} + \alpha}{n_y + V\alpha}$$

same
$$\alpha$$
 for all x_i

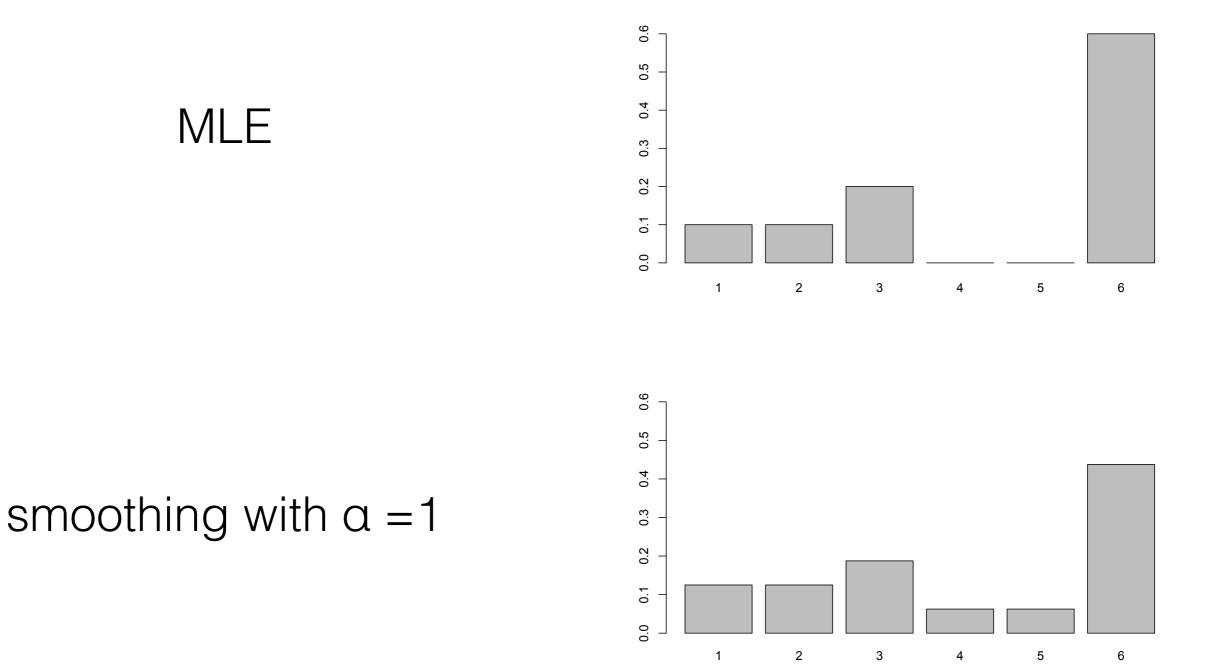
$$P(x_i | y) = \frac{n_{i,y} + a_i}{n_y + \sum_{j=1}^{V} a_j}$$

possibly different α for each x_i

$$P(x_i \mid y) = \frac{n_{i,y}}{n_y}$$

 $n_{i,y}$ = count of word i in class y n_y = number of words in y V = size of vocabulary

Smoothing



Naive Bayes training

Training a Naive Bayes classifier consists of estimating these two quantities from training data for all classes y

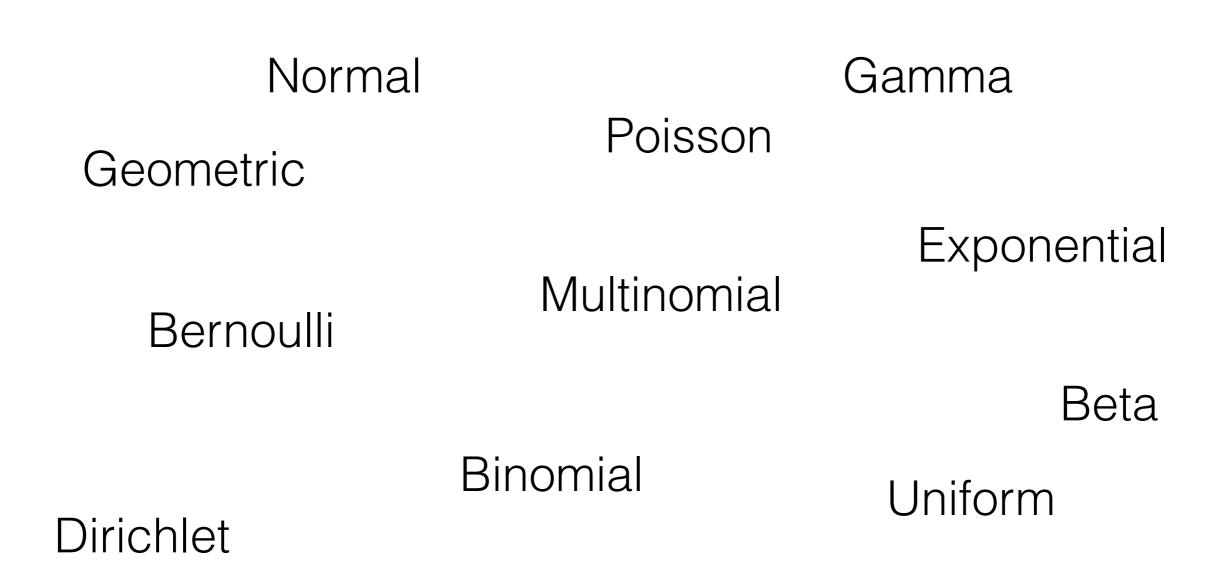
$$P(Y = y | X = x) = \frac{P(Y = y)P(X = x | Y = y)}{\sum_{y} P(Y = y)P(X = x | Y = y)}$$

At test time, use those estimated probabilities to calculate the posterior probability of each class y and select the class with the highest probability

Naive Bayes

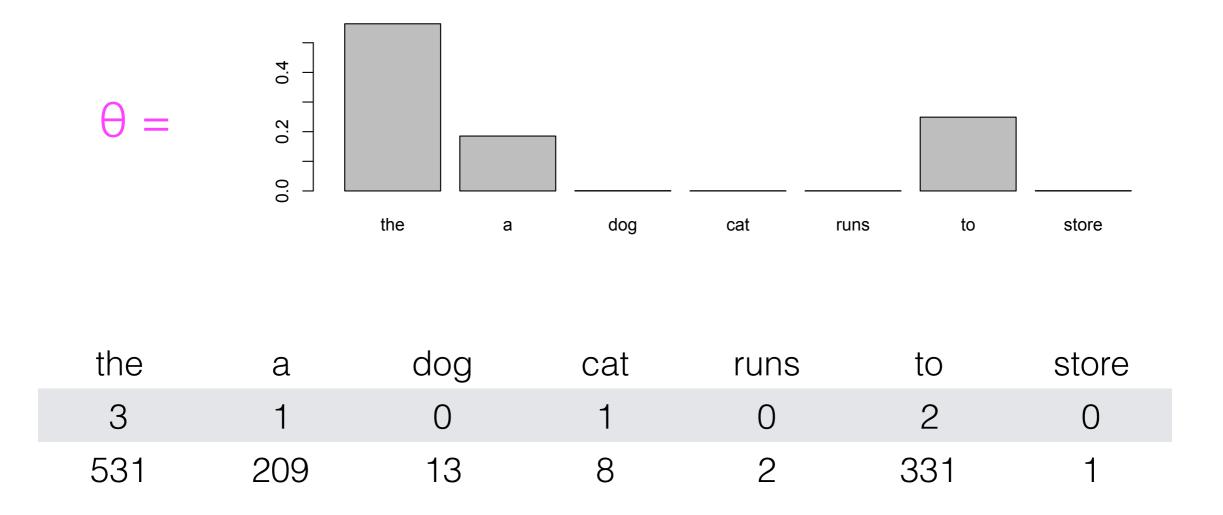
 We've just described Naive Bayes with a multinomial distribution, but any probability distribution can be modeled as well.

Probability distributions



Multinomial

Discrete distribution for modeling count data (e.g., word counts; single parameter Θ



Multinomial

Maximum likelihood parameter estimate

$$\hat{\theta}_i = \frac{n_i}{N}$$

	the	а	dog	cat	runs	to	store
count n	531	209	13	8	2	331	1
θ	0.48	0.19	0.01	0.01	0.00	0.30	0.00

Bernoulli

- Binary event (true or false; {0, 1})
- One parameter: p (probability of an event occurring)

$$P(x = 1 | p) = p$$

 $P(x = 0 | p) = 1 - p$

Examples:

 Probability of a particular feature being true (e.g., self-reported location = Berkeley)

$$\hat{p}_{mle} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

Bernoulli

	X ₁	X2	X3	X4	X 5	X ₆	X7	X8	PMLE
f ₁	1	0	0	0	1	1	0	0	0.375
f_2	0	0	0	0	0	0	1	0	0.125
f ₃	1	1	1	1	1	0	0	1	0.750
f4	1	0	0	1	1	0	0	1	0.500
f_5	0	0	0	0	0	0	0	0	0.000

Bernoulli

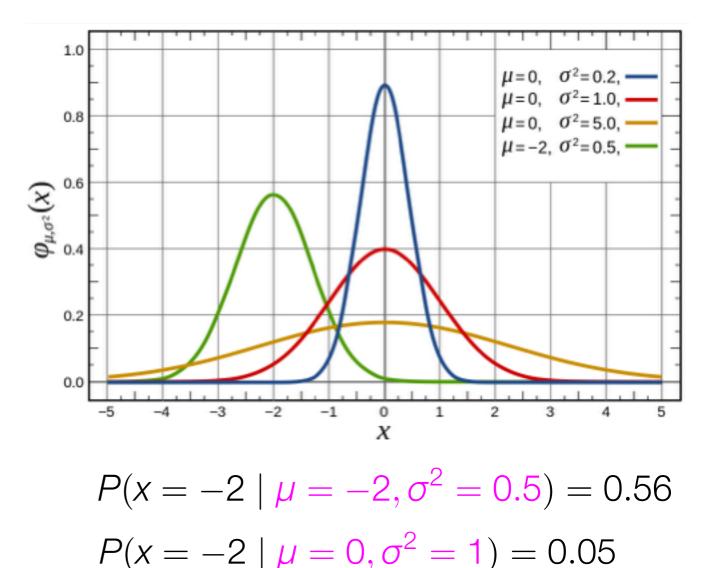
	Republican					Dem	ocrat			
	X1	X 2	X3	X 4	X 5	X6	X7	X 8	PMLE,R	Pmle,d
f ₁	1	0	0	0	1	1	0	0	0.75	0.50
f ₂	0	0	0	0	0	0	1	0	0.00	0.25
f ₃	1	1	1	1	1	0	0	1	1.00	0.50
f ₄	1	0	0	1	1	0	0	1	0.50	0.50
f_5	0	0	0	0	0	0	0	0	0.00	0.00

Normal

- continuous (- ∞ , ∞)
- µ (mean) (-∞, ∞)
- σ^2 (variance) > 0

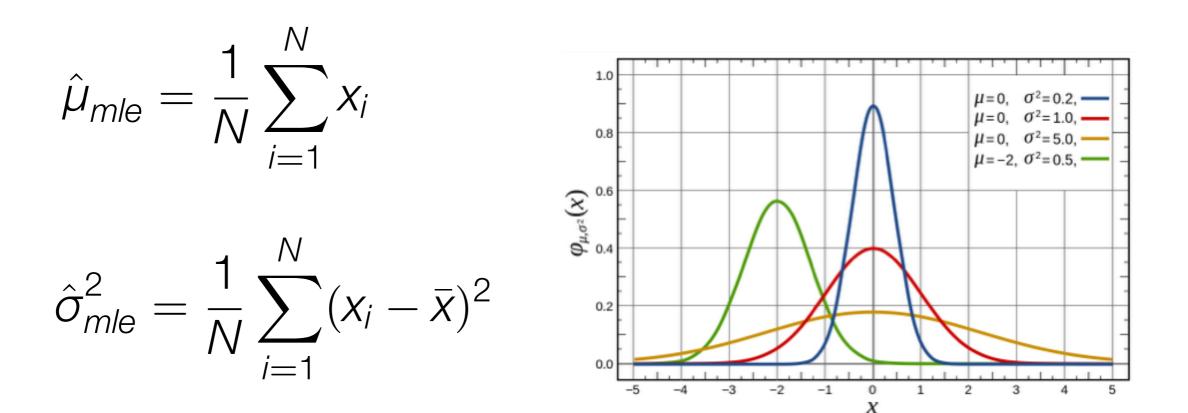
Examples:

- Age
- Height



Normal

Maximum likelihood parameter estimates



Normal

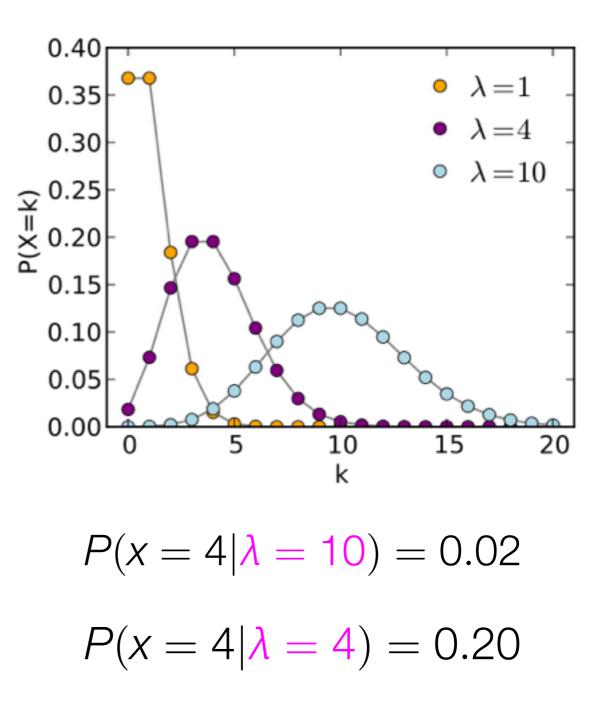
Republican					Democrat					
	X1	X2	X ₃	X 4	X 5	X6	X7	X8	µmle,r	µmle,d
f ₁	3.4	-2.1	5.2	7.6	11.6	9.1	9.7	10.8	3.5	10.3
f ₂	-0.3	8.5	5.6	11.5	5.4	6.2	3.1	12.7	6.3	6.8
f ₃	-0.6	3.7	1.2	5.6	3.4	-4.4	8.0	6.2	2.5	3.3
f4	2.5	6.7	0.5	2.6	13.2	6.1	13.7	7.7	3.1	10.2
f_5	7.0	5.0	5.6	16.3	15.4	14.9	2.3	6.3	8.5	9.7

Poisson

- discrete (0, 1, 2, ...)
- $\lambda > 0$
- Models the number of events within a fixed interval of time

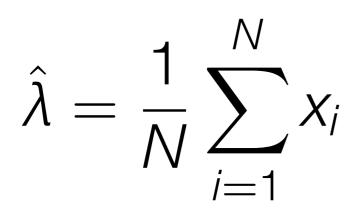
Examples:

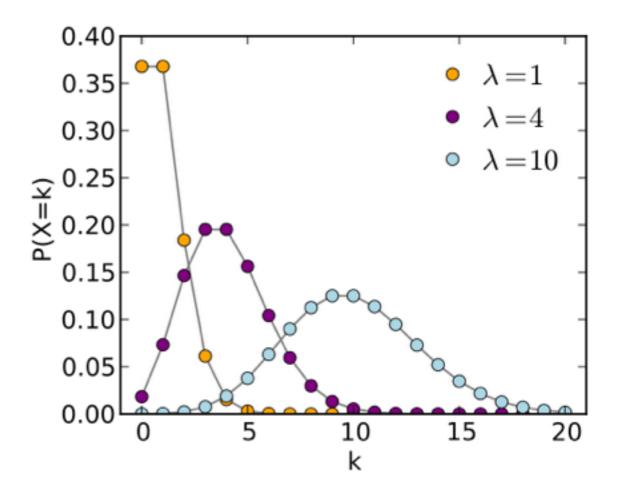
- Number of emails in one hour
- Number of children in family



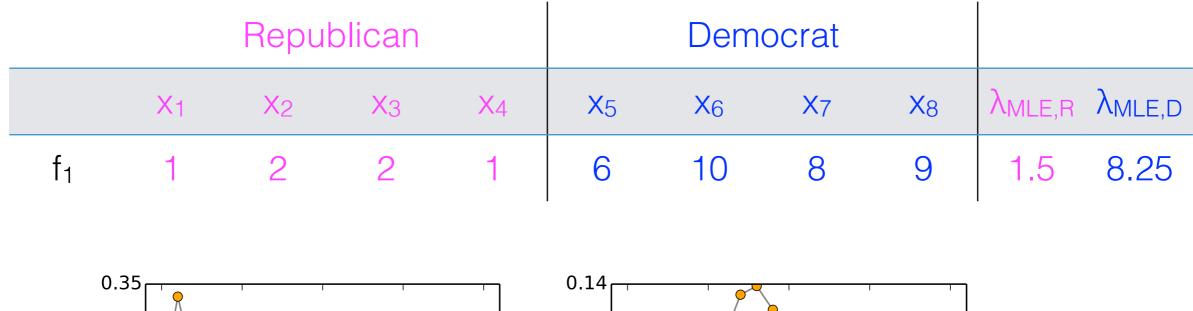
Poisson

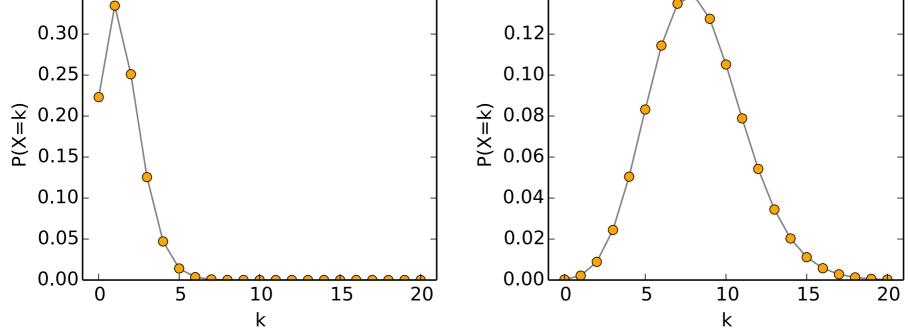
Maximum likelihood parameter estimate



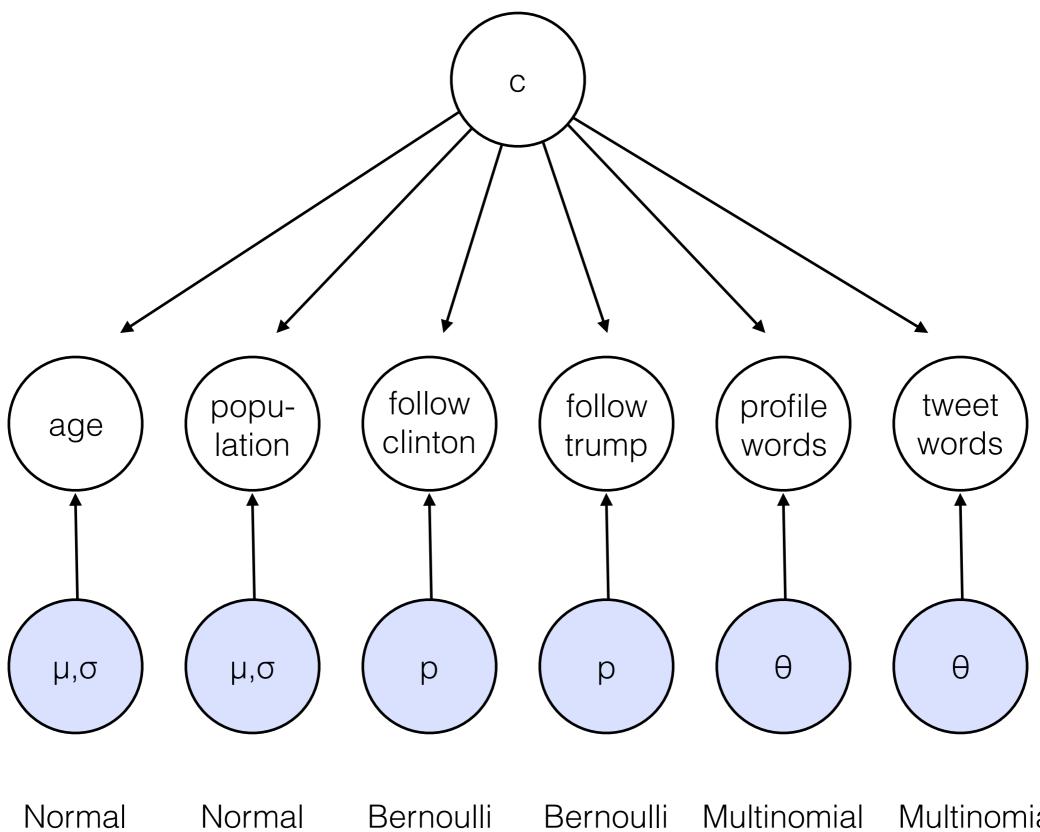


Poisson





Feature	Value	Distribution?
follow clinton	0	
follow trump	0	
age	24	
word counts in profile	Berkeley, liberal, runner	
word counts in profile	the, election, a, data, movies	
population size of your city	116,000	



Normal Bernoulli Bernoulli Multinomial Multinomial

$$P(X \mid c = \text{Dem}) = \prod_{i=1}^{N} P(X_i \mid c = \text{Dem})$$

 $= \operatorname{Norm}(age \mid \mu_{age,dem}, \sigma_{age,dem}^{2})$ $\times \operatorname{Norm}(population \mid \mu_{population,dem}, \sigma_{population,dem}^{2})$ $\times \operatorname{Bernoulli}(followClinton \mid p_{followClinton,dem})$ $\times \operatorname{Bernoulli}(followTrump \mid p_{followTrump,dem})$ $\times \operatorname{Multinomial}(w_{profile} \mid \theta_{profile,dem})$ $\times \operatorname{Multinomial}(w_{tweets} \mid \theta_{tweets,dem})$

$$P(c = \text{Dem} \mid X) = \frac{P(c = \text{Dem}) \times P(X \mid c = \text{Dem})}{P(c = \text{Dem}) \times P(X \mid c = \text{Dem}) + P(c = \text{Rep}) \times P(X \mid c = \text{Rep})}$$

Authorship Attribution

Koppel et al. (2009), Computational Methods in Authorship Attribution (JASIST)

Representation

POSThirty-eight part-of-speech unigrams and 1,000 most common bigrams using the Brill (1992) part-of-speech tagger (purely stylistic)SFLAll 372 nodes in SFL trees for conjunctions, prepositions, pronouns, and modal verbs (purely stylistic)CWThe 1,000 words with highest information gain (Quinlan, 1986) in the training corpus among the 10,000 most common words in the corpusCNGThe 1,000 character trigrams with highest information gain in the training corpus among the 10,000 most common trigrams in the	FW	A list of 512 function words, including conjunctions, prepositions, pronouns, modal verbs, determiners, and numbers (purely stylistic)
SFLand modal verbs (purely stylistic)CWThe 1,000 words with highest information gain (Quinlan, 1986) in the training corpus among the 10,000 most common words in the corpusCNGThe 1,000 character trigrams with highest information gain in the training corpus among the 10,000 most common trigrams in the	POS	
training corpus among the 10,000 most common words in the corpus The 1,000 character trigrams with highest information gain in the training corpus among the 10,000 most common trigrams in the	SFL	
CNG training corpus among the 10,000 most common trigrams in the	CW	
corpus (cf. Keselj, 2003)	CNG	

Models

NB	WEKA's implementation (Witten & Frank, 2000) of Naïve Bayes (Lewis, 1998) with Laplace smoothing
J4.8	WEKA's implementation of the J4.8 decision tree method (Quinlan, 1986) with no pruning
RNW	Our implementation of a version of Littlestone's (1988) Winnow algorithm, generalized to handle real-valued features and more than two classes (Schler, 2007)
BMR	Genkin et al.'s (2006) implementation of Bayesian multiclass regression
SMO	Weka's implementation of Platt's (1998) SMO algorithm for SVM with a linear kernel and default settings

Accuracy

TABLE 2. Accuracy on test set attribution for a variety of feature sets and learning algorithms applied to authorship classification for the e-mail corpus.

	NB	J4.8	RMW	BMR	SMO
Features/learner	(%)	(%)	(%)	(%)	(%)
FW	60.2	58.7	66.1	68.2	63.8
POS	61.0	59.0	66.1	66.3	67.1
FW + POS	65.9	61.6	68.0	67.8	71.7
SFL	57.2	57.2	65.6	67.2	62.7
CW	67.1	66.9	74.9	78.4	74.7
CNG	72.3	65.1	73.1	80.1	74.9
CW+CNG	73.2	68.9	74.2	83.6	78.2

TABLE 4. Accuracy test set attribution for a variety of feature sets and learning algorithms applied to authorship classification for the blog corpus.

Features/learner	NB (%)	J4.8 (%)	RMW (%)	BMR (%)	SMO (%)
FW	38.2	30.3	51.8	63.2	63.2
POS	34.0	30.3	51.0	63.2	60.6
FW + POS	47.0	34.3	62.3	70.3	72.0
SFL	35.4	36.3	61.4	69.2	71.7
CW	56.4	51.0	62.9	72.5	70.5
CNG	65.0	48.9	67.1	80.4	80.9
CW+CNG	69.9	51.6	75.4	86.1	85.7

Homework 2: Validity

HW 2, part I (everyone)

 Pick any of the academic papers assigned throughout this course (i.e., any text except ML and NCM) and discuss the ways in which it establishes (or fails to establish) the nine types of validity outlined in Krippendorff (2004):

- Face validity
- Social validity
- Sampling validity
- Semantic validity
- Structural validity
- Functional validity
- Convergence validity
- Discriminant validity
- Predictive validity

Deliverable: one-page paper

HW 2, part IIa (implementation)

- The permutation test is a robust hypothesis test that doesn't require the parametric or large-sample assumptions of classical tests.
- The GitHub repository contains a dataset mapping movies (featurized through their genres and major actors who performed in them) to a binary decision of whether or not it was among the 25% highest grossing movies in that set.
- For each of the features x, consider the hypothesis "Movies with x are more likely to have a higher box office than those that do not." Code and execute a permutation test evaluating this hypothesis. Can the null hypothesis (that movies featuring x are not likely to have a higher box office than those that do not) be rejected with p < 0.01?

HW 2, part IIb (critique)

- The nine forms of validity outlined above represent a detailed taxonomy of the different ways in which an analysis can be judged for the extend which it is valid. What other possible forms of validity are missing from this taxonomy that should be represented within it? Present an argument for a single form of validity—a.) why it captures an important dimension that should be assessed, b.) why you believe it's missing from Krippendorff's taxonomy, and c.) tangible ways in which an analysis could be assessed according to this dimension.
- Deliverable: one-page paper (single-spaced)